Risk Margin for the Runoff of Non-Life Insurance Reserves

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Insurance contracts generate (random) insurance payment cash flows.

Aim:

1. **Predict and value** these insurance payment cash flows!

These predictions and valuations should always be based on the latest information available.
Reserves and provisions

- Prediction of the outstanding liabilities gives the (claims) reserves or the (claims) provisions.

- These reserves (or provisions)
  - should suffice to meet all future payments
    \[\implies \text{reserves and solvency};\]
  - are the basis for future premium calculations;
  - determine the risk management process.

- These reserves are the most important insurance position at all.

What are the main requirements that these reserves should fulfill?
Full balance sheet approach

- Balance sheet should be valued in a **market-consistent** way:
  - market values where available;
  - marked-to-model approach otherwise.

- **Insurance liabilities:**
  No market values.

- Therefore for reserves:
  **Market-consistent prediction of outstanding insurance liabilities in a marked-to-model approach.**

- What does this exactly mean?
Technical provisions

- Solvency II Directive 2009/138/EC:
  Insurance liabilities should be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arm’s length transaction.

- The resulting amount is called technical provisions.

- The technical provisions are the sum of the best-estimate reserves and the risk margin.

- What are best-estimate reserves? Why a risk margin?

\[ \text{deterministic best-estimate reserves} \iff \text{stochastic claims payments} \]
Best-estimate reserves for outstanding liabilities

“The best-estimate should correspond to the probability weighted average of future cash flows taking account of time value of money.”

Mathematical model: \((\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t\in\mathbb{N}})\) filtered probability space:

- \(\mathcal{F}_t\) information available at time \(t \in \mathbb{N}\);
- \(\varphi = (\varphi_t)_{t\in\mathbb{N}}\) stochastic discount function (financial deflator);
- \(X = (X_t)_{t\in\mathbb{N}}\) insurance liability cash flow, \((\mathcal{F}_t)_{t\in\mathbb{N}}\)-adapted.

**Best-estimate reserves** at time \(k \in \mathbb{N}\) for liabilities \((X_t)_{t>k}\) (see [3])

\[
R_k(X) = \sum_{t>k} \mathbb{E} \left[ \frac{\varphi_t}{\varphi_k} X_t \bigg| \mathcal{F}_k \right].
\]
Stochastic discounting of best-estimate reserves

- Find appropriate stochastic model \((\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t \in \mathbb{N}})\) and
  - \(\varphi = (\varphi_t)_{t \in \mathbb{N}}\) stochastic discount function (financial deflator),
  - \(X = (X_t)_{t \in \mathbb{N}}\) insurance liability cash flow,

and calculate best-estimate reserves at time \(k \in \mathbb{N}\)

\[
R_k(X) = \sum_{t > k} \mathbb{E} \left[ \frac{\varphi_t}{\varphi_k} X_t \mid \mathcal{F}_k \right].
\]

- In general, with \((r_t^{(k)})_{t \geq 0}\) risk-free term structure at time \(k\),

\[
R_k(X) \neq \sum_{t > k} \frac{1}{\left(1 + r_{t-k}^{(k)}\right)^{t-k}} \mathbb{E} \left[ X_t \mid \mathcal{F}_k \right],
\]

due to options, guarantees, inflation, etc.
Risk margin or market-value margin MVM

- How reliable is the prediction $R_k(X)$ for $(X_t)_{t>k}$?

- A risk averse agent asks for a risk margin (market-value margin) for possible shortfalls in this prediction.

- The technical provisions (market-consistent value) for the outstanding insurance liabilities are then given by

  $$R_k^*(X) = R_k(X) + \text{MVM}_k(X).$$

- How should we calculate this risk margin $\text{MVM}_k(X)$?
**Solvency at time $k$**

Solvency is given at time $k$ iff:

1. asset values *cover* technical provisions $R^*_k(X)$ at time $k$, and
2. the possibility of an asset deficit $AD_{k+1} > 0$ at time $k+1$ is sufficiently small (measured by an appropriate risk measure).
Conclusions for solvency calculation

We need a **stochastic model** that allows for:

1. calculation of best-estimate reserves $\mathcal{R}_k(X)$ at time $k$;

2. calculation of risk margin $\text{MVM}_k(X)$ at time $k$;

3. modeling of asset deficit $\text{AD}_{k+1}$ at time $k + 1$.

**Note:** Everything holds true for life and non-life insurance.
Risk margin in non-life insurance

- We give 3 different approaches for the calculation of the risk margin.

- In non-life insurance one often assumes that claims payments $X$ are independent from financial market developments. This implies

$$\mathcal{R}_k(X) = \sum_{t>k} \mathbb{E} [X_t | \mathcal{F}_k] \ P(k, t),$$

with $P(t, k)$ price of the zero coupon bond with maturity $t$ at time $k$.

- The claims development result (CDR) at time $k + 1$ is given by

$$\text{CDR}(k + 1) = \left( \sum_{t>k} \mathbb{E} [X_t | \mathcal{F}_k] \ P(k + 1, t) \right) - (X_{k+1} + \mathcal{R}_{k+1}(X)).$$
Claims development result

The CDR at time $k+1$

\[
\text{CDR}(k+1) = \left( \sum_{t>k} \mathbb{E} \left[ X_t \middle| \mathcal{F}_k \right] P(k+1, t) \right) - (X_{k+1} + \mathcal{R}_{k+1}(X))
\]

considers the update of information $\mathcal{F}_k \mapsto \mathcal{F}_{k+1}$:

- $\text{CDR}(k+1) < 0$: additional capital is needed;
- $\text{CDR}(k+1) > 0$: we have a gain in the P&L statement.
Risk margin: approach 1

Cost-of-capital risk margin:

1. Calculate the solvency capital requirement (SCR) for possible shortfalls in this CDR position.

   \[ \Rightarrow \text{This provides risk measure } \rho_k \text{ for accounting year } k + 1. \]

2. The risk margin should be related to this SCR \( \rho_k \).
Market-value margin 1 (current solvency practice)

The first cost-of-capital (CoC) approach defines the risk margin as

\[
MVM_{1}^{(1)}(X) = r_{CoC} \cdot \sum_{t>k} w_t \cdot \rho_t,
\]

where

- \( \rho_t \) risk measure (SCR) for possible shortfalls in \( \text{CDR}(k + 1) \);
- \( r_{CoC} \) cost-of-capital rate \( > r_{0}^{(k)} \) (risk-free rate at time \( k \));
- \( (w_{k+1}, w_{k+2}, w_{k+3}, \ldots) \) expected runoff of the outstanding liabilities \( (X_{k+1}, X_{k+2}, X_{k+3}, \ldots) \) at time \( k \).

**Interpretation.** The risk margin from this CoC approach should reflect the **reward for risk bearing**, i.e. an investor provides the SCRs \( w_t \cdot \rho_t \) and therefore receives a rate of return \( r_{CoC} > r_{0}^{(k)} \) on these SCRs.
Difficulties with the market-value margin 1

\[ \text{MVM}^{(1)}_k(X) = r_{\text{CoC}} \cdot \sum_{t>k} w_t \cdot \rho_k, \]

1. Choice of risk measure \( \rho_k \) (SCR):
   - runoff or going-concern view?
   - stand-alone or diversified?
   - per line-of-business or whole insurance portfolio?

2. Choice of \( r_{\text{CoC}} \)? Is \( r_{\text{CoC}} = r^{(k)}_0 + 6\% \) appropriate?

3. \( w_t \cdot \rho_k \) is not a risk-based approximation to the SCRs \( \rho_t \) in accounting years \( t > k \).
Risk margin: approach 2 (Salzmann-W. [1])

For simplicity, we choose nominal reserves, i.e. $P(k, t) \equiv 1$.

Then

$$\text{CDR}(k + 1) = \sum_{t > k} \mathbb{E} \left[ X_t | F_k \right] - \sum_{t > k} \mathbb{E} \left[ X_t | F_{k+1} \right].$$

This implies for $t > k$

$$\mathbb{E} \left[ \text{CDR}(t) | F_k \right] = 0,$$

and, moreover,

$\text{CDR}(k + 1), \text{CDR}(k + 2), \ldots$ are uncorrelated (not independent).

This follows because successive best-estimate predictions are martingales.
Uncorrelatedness provides the total prediction variance decomposition

\[
\text{Var} \left( \sum_{t>k} \text{CDR}(t) \mid \mathcal{F}_k \right) = \sum_{t>k} \text{Var} \left( \text{CDR}(t) \mid \mathcal{F}_k \right).
\]

Formula (1) gives a risk-based allocation of the total uncertainty measured by the prediction variance to individual accounting years.

In many models we can explicitly calculate \( \text{Var} \left( \text{CDR}(t) \mid \mathcal{F}_k \right) \), e.g. \( \Gamma-\Gamma \) Bayes chain ladder model of Salzmann-W. [1].
Market-value margin 2 (split of total uncertainty)

The second cost-of-capital (CoC) approach defines the risk margin as

\[
\text{MVM}_{k}^{(2)}(X) = r_{\text{CoC}} \cdot \sum_{t>k} \Phi \cdot \text{Var}(\text{CDR}(t)|\mathcal{F}_k)^{1/2},
\]

where

- \( \rho_t = \Phi \cdot \text{Var}(\text{CDR}(t)|\mathcal{F}_k)^{1/2} \) standard deviation risk measure for possible shortfalls in \( \text{CDR}(t) \) on security level \( \Phi > 0 \);
- \( r_{\text{CoC}} \) cost-of-capital rate > \( r_0^{(k)} \) (risk-free rate at time \( k \)).

Remarks.

- This provides a risk-adjusted market-value margin.
- Other (multi-period) risk measures are too involved and do not lead to applicable solutions (nested simulations).
Risk margin: approach 3 (economic approach)

**Idea:** The technical provisions $R_k^*(X)$ should be a market-consistent price for the outstanding insurance liabilities:

- a rational investor calculates under risk aversion a margin for non-hedgeable (insurance technical) risks.
- In economic theory this is usually done with utility functions and/or with probability distortions.

For non-life claims reserving probability distortions are a feasible way, see W. et al. [3], Section 2.6.
Market-value margin 3 (chain-ladder framework 1/2)

For a particular chain-ladder claims reserving model (W.-Merz [4]):

- **Best-estimate reserves:**

  \[ R_k(X) = C_k \left( \prod_{t>k} f_t - 1 \right), \]

  where \( C_k \) are the cumulative payments at time \( k \) and \( f_t \) are the so-called chain-ladder factors.

- **Technical provisions:**

  \[ R_{k}^{(*)}(X) = C_k \left( \prod_{t>k} f_t^{(*)} - 1 \right), \]

  where \( f_t^{(*)} \) are the **risk-adjusted** chain-ladder factors.
Market-value margin 3 (chain-ladder framework 2/2)

- **Risk margin:**

\[
MVM_{k}^{(3)}(X) = \mathcal{R}_{k}^{(*)}(X) - \mathcal{R}_{k}(X) = C_{k} \left( \prod_{t > k} f_{t}^{(*)} - \prod_{t > k} f_{t} \right).
\]

- **Risk-adjusted chain-ladder factors:** a sensible choice is

\[
f_{t}^{(*)} = (f_{t} - 1) \exp \{ h_{t}(\alpha) \} + 1 > f_{t},
\]

for \( h_{t}(\alpha) > 0 \) a positive function of the risk aversion parameter \( \alpha \).
Case studies: private liability insurance

Expected runoff pattern of risk margins $\text{MVM}^{(i)}_k(\mathbf{X})$ for approaches $i = 1, 2, 3$. 
Case studies: life-time annuity

Expected runoff pattern of risk margins $\text{MVM}_k(\mathbf{X})^{(i)}$ for approaches $i = 1, 3$. 
Case studies: motor third party liability insurance

Expected runoff pattern of risk margins $\text{MVM}_k^{(i)}(X)$ for approaches $i = 1, 2$. 
Case studies: general liability insurance

Expected runoff pattern of risk margins $\text{MVM}^{(i)}_k(X)$ for approaches $i = 1, 2$. 
Case studies: property insurance

Expected runoff pattern of risk margins $\text{MVM}^{(i)}_{k}(\mathbf{X})$ for approaches $i = 1, 2$. 
Case studies: health insurance

Expected runoff pattern of risk margins $\text{MVM}_k^{(i)}(X)$ for approaches $i = 1, 2$. 
References


